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1. Bernoulli.

a) These are not Bernoulli trials. The possible outcomes are 1, 2, 3, 4, 5, and 6. There are more than two possible outcomes.

b) These may be considered Bernoulli trials. There are only two possible outcomes, Type A and not Type A. Assuming the 120 donors are representative of the population, the probability of having Type A blood is 43%. The trials are not independent, because the population is finite, but the 120 donors represent less than 10% of all possible donors.

c) These are not Bernoulli trials. The probability of getting a heart changes as cards are dealt without replacement.

d) These are not Bernoulli trials. We are sampling without replacement, so the trials are not independent. Samples without replacement may be considered Bernoulli trials if the sample size is less than 10% of the population, but 500 is more than 10% of 3000.

e) These may be considered Bernoulli trials. There are only two possible outcomes, sealed properly and not sealed properly. The probability that a package is unsealed is constant, at about 10%, as long as the packages checked are a representative sample of all packages. Finally, the trials are not independent, since the total number of packages is finite, but the 24 packages checked probably represent less than 10% of the packages.

7. Hoops.

The player's shots may be considered Bernoulli trials. There are only two possible outcomes (make or miss), the probability of making a shot is constant (80%), and the shots are independent of one another (making, or missing, a shot does not affect the probability of making the next).

Let X = the number of shots until the first missed shot.
Let Y = the number of shots until the first made shot.

Since these problems deal with shooting until the first miss (or until the first made shot), a geometric model, either $Geom(0.8)$ or $Geom(0.2)$, is appropriate.

a) Use $Geom(0.2)$. $P(X=5)=(0.8)^4(0.2)=0.08192$ (Four shots made, followed by a miss.)

b) Use $Geom(0.8)$. $P(Y=4)=(0.2)^3(0.8)=0.0064$ (Three misses, then a made shot.)

c) Use $Geom(0.8)$. $P(Y=1)+P(Y=2)+P(Y=3)=(0.8)+(0.2)(0.8)+(0.2)^2(0.8)=0.992$

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11. Blood.

These may be considered Bernoulli trials. There are only two possible outcomes, Type AB and not Type AB. Provided that the donors are representative of the population, the probability of having Type AB blood is constant at 4%. The trials are not independent, since the population is finite, but we are selecting fewer than 10% of all potential donors. Since we are selecting people until the first success, the model $Geom(0.04)$ may be used.

Let X = the number of donors until the first Type AB donor is found.

a) $E(X) = \frac{1}{p} = \frac{1}{0.04} = 25$ people We expect the 25th person to be the first Type AB donor.

b)

P(a Type AB donor among the first 5 people checked)

$= P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= (0.04) + (0.96)(0.04) + (0.96)^2(0.04) + (0.96)^3(0.04) + (0.96)^4(0.04) \approx 0.185$

c)

P(a Type AB donor among the first 6 people checked)

$= P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6)$

$= (0.04) + (0.96)(0.04) + (0.96)^2(0.04) + (0.96)^3(0.04) + (0.96)^4(0.04) + (0.96)^5(0.04) \approx 0.217$

d) P(no Type AB donor before the 10th person checked) $= P(X > 9) = (0.96)^9 \approx 0.693$

This one is a bit tricky. There is no implication that we actually find a donor on the 10th trial. We only care that nine trials passed with no Type AB donor.

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13. Lefties.

These may be considered Bernoulli trials. There are only two possible outcomes, left-handed and not left-handed. Since people are selected at random, the probability of being left-handed is constant at about 13%. The trials are not independent, since the population is finite, but a sample of 5 people is certainly fewer than 10% of all people.

Let X = the number of people checked until the first lefty is discovered.
Let Y = the number of lefties among $n = 5$.

a) Use $Geom(0.13)$.
$P(\text{first lefty is the fifth person}) = P(X = 5) = (0.87)^4(0.13) \approx 0.0745$

b) Use $Binom(5,0.13)$.

$$\begin{aligned}P(\text{some lefties among the 5 people}) &= 1 - P(\text{no lefties among the first 5 people})\\ &= 1 - P(Y = 0)\\ &= 1 - \binom{5}{0}(0.13)^0(0.87)^5\\ &\approx 0.502\end{aligned}$$

c) Use $Geom(0.13)$.
$P(\text{first lefty is second or third person}) = P(X = 2) + P(X = 3) = (0.87)(0.13) + (0.87)^2(0.13) \approx 0.211$

d) Use $Binom(5,0.13)$.
$P(\text{exactly 3 lefties in the group}) = P(Y = 3) = \binom{5}{3}(0.13)^3(0.87)^2 \approx 0.0166$

e) Use $Binom(5,0.13)$.

$$\begin{aligned}P(\text{at least 3 lefties in the group}) &= P(Y = 3) + P(Y = 4) + P(Y = 5)\\ &= \binom{5}{3}(0.13)^3(0.87)^2 + \binom{5}{4}(0.13)^4(0.87)^1 + \binom{5}{5}(0.13)^5(0.87)^0\\ &\approx 0.0179\end{aligned}$$

f) Use $Binom(5,0.13)$.

$$\begin{aligned}P(\text{at most 3 lefties in the group}) &= P(Y = 0) + P(Y = 1) + P(Y = 2) + P(Y = 3)\\ &= \binom{5}{0}(0.13)^0(0.87)^5 + \binom{5}{1}(0.13)^1(0.87)^4\\ &\quad + \binom{5}{2}(0.13)^2(0.87)^3 + \binom{5}{3}(0.13)^3(0.87)^2\\ &\approx 0.9987\end{aligned}$$

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15. Lefties redux.

a) In Exercise 9, we determined that the selection of lefties could be considered Bernoulli trials. Since our group consists of 5 people, use *Binom*(5,0.13).

Let Y = the number of lefties among $n = 5$.

$E(Y) = np = 5(0.13) = 0.65$ lefties

b) $SD(Y) = \sqrt{npq} = \sqrt{5(0.13)(0.87)} \approx 0.75$ lefties

c) Use *Geom*(0.13). Let X = the number of people checked until the first lefty is discovered.

$E(X) = \frac{1}{p} = \frac{1}{0.13} \approx 7.69$ people

19. Tennis, anyone?

The first serves can be considered Bernoulli trials. There are only two possible outcomes, successful and unsuccessful. The probability of any first serve being good is given as $p = 0.70$. Finally, we are assuming that each serve is independent of the others. Since she is serving 6 times, use *Binom*(6,0.70).

Let X = the number of successful serves in $n = 6$ first serves.

a)

$$P(\text{all six serves in}) = P(X = 6) = \binom{6}{6}(0.70)^6(0.30)^0 \approx 0.118$$

b)

$$P(\text{exactly four serves in}) = P(X = 4) = \binom{6}{4}(0.70)^4(0.30)^2 \approx 0.324$$

c)

$$P(\text{at least four serves in}) = P(X=4) + P(X=5) + P(X=6) = \binom{6}{4}(0.70)^4(0.30)^2 + \binom{6}{5}(0.70)^5(0.30)^1 + \binom{6}{6}(0.70)^6(0.30)^0 \approx 0.744$$

d)

$$P(\text{no more than four serves in}) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$= \binom{6}{0}(0.70)^0(0.30)^6 + \binom{6}{1}(0.70)^1(0.30)^5 + \binom{6}{2}(0.70)^2(0.30)^4 + \binom{6}{3}(0.70)^3(0.30)^3 + \binom{6}{4}(0.70)^4(0.30)^2$$

$$\approx 0.580$$

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21. And more tennis.

The first serves can be considered Bernoulli trials. There are only two possible outcomes, successful and unsuccessful. The probability of any first serve being good is given as $p = 0.70$. Finally, we are assuming that each serve is independent of the others. Since she is serving 80 times, use *Binom*(80,0.70).

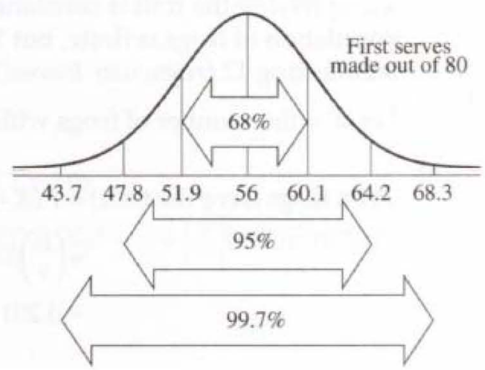
Let X = the number of successful serves in $n = 80$ first serves.

a) $E(X) = np = 80(0.70) = 56$ first serves in.

$SD(X) = \sqrt{npq} = \sqrt{80(0.70)(0.30)} \approx 4.10$ first serves in.

b) Since $np = 56$ and $nq = 24$ are both greater than 10, *Binom*(80,0.70) may be approximated by the Normal model, $N(56, 4.10)$.

c) According to the Normal model, in matches with 80 serves, she is expected to make between 51.9 and 60.1 first serves approximately 68% of the time, between 47.8 and 64.2 first serves approximately 95% of the time, and between 43.7 and 68.3 first serves approximately 99.7% of the time.



d) **Using *Binom*(80, 0.70):**

$$\begin{aligned}
P(\text{at least 65 first serves}) &= P(X \geq 65) \\
&= P(X = 65) + P(X = 66) + \ldots + P(X = 80) \\
&= \binom{80}{65}(0.70)^{65}(0.30)^{15} + \binom{80}{66}(0.70)^{66}(0.30)^{14} + \ldots + \binom{80}{80}(0.70)^{80}(0.30)^{0} \\
&= 0.0161
\end{aligned}$$

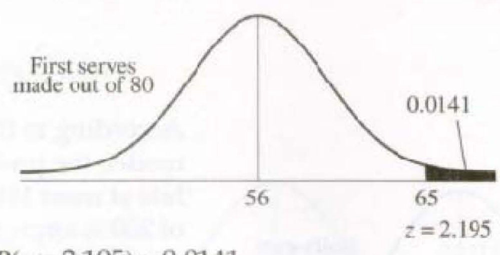
According to the Binomial model, the probability that she makes at least 65 first serves out of 80 is approximately 0.0161.

Using $N(56, 4.10)$:

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{65 - 56}{4.10}$$

$$z \approx 2.195$$



$P(X \geq 65) \approx P(z > 2.195) \approx 0.0141$

According to the Normal model, the probability that she makes at least 65 first serves out of 80 is approximately 0.0141.

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25. Lefties again.

Let X = the number of righties among a class of n = 188 students.

Using *Binom*(188, 0.87):

These may be considered Bernoulli trials. There are only two possible outcomes, right-handed and not right-handed. The probability of being right-handed is assumed to be constant at about 87%. The trials are not independent, since the population is finite, but a sample of 188 students is certainly fewer than 10% of all people. Therefore, the number of righties in a class of 188 students may be modeled by *Binom*(188, 0.87).

If there are 171 or more righties in the class, some righties have to use a left-handed desk.

$$\begin{aligned} P(\text{at least 171 righties}) &= P(X \geq 171) \\ &= P(X = 171) + \ldots + P(X = 188) \\ &= \binom{188}{171}(0.87)^{171}(0.13)^{17} + \ldots + \binom{188}{188}(0.87)^{188}(0.13)^{0} \\ &\approx 0.061 \end{aligned}$$

According to the binomial model, the probability that a right-handed student has to use a left-handed desk is approximately 0.061.

Using $N(163.56, 4.61)$:

$E(X) = np = 188(0.87) = 163.56$ righties.

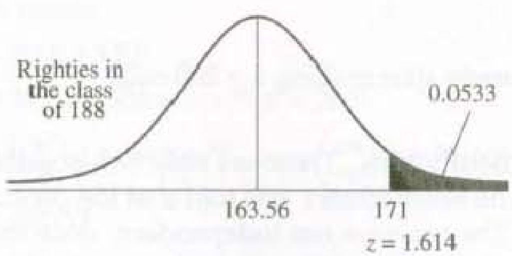
$SD(X) = \sqrt{npq} = \sqrt{188(0.87)(0.13)} \approx 4.61$ righties.

Since $np = 163.56$ and $nq = 24.44$ are both greater than 10, $Binom(188, 0.87)$ may be approximated by the Normal model, $N(163.56, 4.61)$.

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{171 - 163.56}{4.61}$$

$$z \approx 1.614$$



$P(X \geq 171) \approx P(z > 1.614) \approx 0.053$

According to the Normal model, the probability that there are at least 171 righties in the class of 188 is approximately 0.0533.